Representing Text with Vectors

Machine Learning for Natural Language Processing, ENSAE 2022

Lecture 2

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Lectures Outline

1. The Basics of Natural Language Processing (February 1st)
2. **Representing Text with Vectors (February 1st)**
3. Deep Learning Methods for NLP (February 8th)
4. Language Modeling (February 8th)
5. Sequence Labelling (Sequence Classification) (February 15th)
6. Sequence Generation Tasks (February 15th)

Today Lecture Outline

- **Representing Words** in Vectors
- **Representing Documents** in Vectors

Representation Techniques

→ **Hand-Crafted Feature-Based** Representation
→ **Count-Based** Representation
→ **Prediction-Based** Representation

Framework

We assume:

- A **token** is the basic unit of discrete data, defined to be an item from a vocabulary indexed by 1, ..., V.
- A **document** is a sequence of N words denoted by **d = (w1,w2, ...,wN)**, where wn is the Nth word in the sequence.
- A **corpus** is a collection of M documents denoted by D = (d1, d2, ..., dM)

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In this lecture, a token will be **a word**

What is a word?

There are many ways to define a word based on what aspect of language we consider (typography, syntax, semantics…)

Definition (Semantic):

*Words are **the smallest linguistic expressions** that are **conventionally** associated with a **non-compositional meaning** and can be articulated in isolation to convey semantic content.**

*Stanford Encyclopedia of Philosophy

Objective

Given a vocabulary w1,..,wV and a corpus D, our goal is to associate each word with a representation?

What do we want from this representation?

- identify a word (bijection)
- capture the similarities of words (based on morphology, syntax, semantics,...)
- Help us solve downstream tasks

NB: Vector-based representations of text are called ***embedding***

1-Hot Encoding

Traditional way to represent words **as atomic symbols** with a unique integer associated with each word:

{1=movie, 2=hotel, 3=apple, 4=movies, 5=art}

Equivalent to represent words as 1-hot vectors:

movie = [1, 0, 0, 0, 0]
hotel = [0, 1, 0, 0, 0]
...
art = [0, 0, 0, 0, 1]

1-Hot Encoding

Most basic representation of any textual unit in NLP. Always start with it.

Implicit assumption: word vectors are an **orthonormal basis**

- orthogonal
- normalized

Problem 1: **Not very informative**

➔ Weird to consider "movie" and "movies" as independent entities or to consider all words equidistant:

$$\| house - home \| = \| house - car \|$$

Problem 2: Polysemy

➔ Should the Mouse of a computer get the same vector a the mouse animal?

Hand-Crafted Feature Representation

Example of potential features:

- Morphology: prefix, suffix, stem...
- Grammar: part of speech, gender, number,...
- Shape: capitalization, digit, hyphen

Those features can be defined based on **relations** to other words

- Synonyms of…
- Hypernyms of…
- Antonyms of...

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We present one popular hand-crafted semantically based representation of words ⇒ the WordNet

WordNet

Definition: a (word) sense is a discrete representation **of one aspect of the meaning of a word**

WordNet is a large lexical database of **word senses** for English and other languages

WordNet

- Word types are grouped into (cognitive) synonym sets: **synsets**
  S09293800={*Earth,earth,world,globe*}

- **Polysemous** words: assigned to **different synsets**
  S14867162 ={*earth,ground*}

- Contains glosses for synsets:
  the 3rd planet from the sun; the planet we live on

- **Noun/verb synsets:** organized in **hierarchy**, capturing IS-A relation
  apple IS-A *fruit*

WordNet

X is a **hyponym** of Y if **X is an instance of Y**:
cat is a hyponym of animal

X is a **hypernym** of Y if **Y is an instance of X**:
animal is a hypernym of cat

X and Y are **co-hyponyms** if they have the **same hypernym**:
cat and dog are co-hyponyms

X is a **meronym** of Y if **X is a part of Y**:
wheel is a meronym of car

X is a **holonym** of Y if **Y is a part of X**:
car is a holonym of wheel

WordNet

Similarity $$\text{sim}(S_1, S_2) = \frac{1}{\text{length}(\text{path}(S_1, S_2))}$$ ***t:***

Idea: *The shorter the* ***hypernym/hyponym*** *path from one synset to another the higher is the similarity*

Similarity **b** $$\text{sim}(w_1, w_2) = \max_{\substack{S_1, S_2 \\ w_1 \in S_1 \\ w_2 \in S_2}} \text{sim}(S_1, S_2)$$ **ds:**

Example: sim(*dog*, *cat*) = ? 👉 notebook

Hand-Crafted Representations: Limits

- Requires **a lot of human annotations**
- **Subjectivity** of the annotators
- **Does not adapt** to new words (languages are not stationary!): *Mocktail, Guac, Fave, Biohacking* were added to the Merriam-Webster Dictionary in 2018

➔ It **does not scale** easily to new languages, new concepts, new words...

How to Infer "Good" Representations with Data?

Distributional Hypothesis

You shall know a word by the company it keeps" Firth (1957)

Idea: Model the *context* of a word to build **its vectorial representation**

Example: What is the meaning of "**Bardiwac**" ?

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→ bardiwac is a heavy red alcoholic beverage made from grapes

Distributional word representation in a nutshell

1. Define what is ***the context*** of a word

2. **Count** how many times each target word occurs in this context

3. Build vectors out of (a function of) these context occurrence counts

$$x_w = f(w, Context(w))$$

How to define ***"the context"*** of a word?

It can be defined as

- **The surrounding words** (left and right words)
- All the other words of the sentence/the paragraph
- All the words **after preprocessing and filtering-out some words**

How to Model the Context to get

$$x_w = f(w, Context(w))$$

Approach 1: Count-Based

1. Measure frequency of words in the context for each word in the vocabulary
2. Define vector representations based on those frequency

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Approach 2: Prediction Based

Counting the Occurences of the words in the context of dog

The dog barked in the park.
The owner of the dog put him
on the leash since he barked.

co-occurence # dog	
barked	++
park	+
owner	+
leash	+

Co-Occurrence Matrix

	leash	walk	run	owner	pet	barked
dog	3	5	2	5	3	2
cat	0	3	3	2	3	0
lion	0	3	2	0	1	0
light	0	0	0	0	0	0
bark	1	0	0	2	1	0
car	0	0	1	3	0	0

Define vector representation based on the Co-Occurrence

	leash	walk	run	owner	pet	barked	the
dog	3	5	2	5	3	2	8
lion	0	3	2	0	1	0	6
light	0	0	0	0	0	0	5
bark	1	0	0	2	1	0	0
car	0	0	1	3	0	0	3

- **Naïve Approach: Take the row of the co-occurrence matrix**

Define vector representation based on the Co-Occurrence

	leash	walk	run	owner	pet	barked	the
dog	3	5	2	5	3	2	8
lion	0	3	2	0	1	0	6
light	0	0	0	0	0	0	5
bark	1	0	0	2	1	0	0
car	0	0	1	3	0	0	3

Limits: 👉 notebook

- Representations depends **on the size of the corpus**
- Frequent words impacts a lot the representations
- Representations very sensitive to change in very infrequent words

Solution: Pointwise Mutual Information (PMI)

Idea: Instead of absolute co-occurrence statistics, use probability (relative) of co-occurrences

$$PMI(w_1, w_2) = log \frac{P(w_1, w_2)}{P(w_1)P(w_2)}$$

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Intuition

- **The more dependent *dog* and *cat*** the closer P(dog, cat) is from P(dog)P(cat) **the smaller the PMI**

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$$PMI(w_1, w_2) = log \frac{\frac{1}{n_{pairs}}\#\{(w_1, w_2)\}}{\frac{1}{n_{word}}\#\{w_1\}\frac{1}{n_{word}}\#\{w_2\})}$$

Pointwise Mutual Information (PMI)

	leash	walk	run	owner	pet	barked	the
dog	2.75	2.24	3.16	2.24	2.75	3.16	1.77
lion	0	2.75	3.16	0	3.85	0	2.06
car	0	0	3.85	2.75	0	0	2.75

Word embedding vectors are the row of the PMI matrix

- We take usually take the Positive PMI (assigned to 0 when negative) + Smooth unobserve pairs (Laplace smoothing: add 1)
- Does not depend on size of the corpus (the PMI is **normalized**)
- Much less sensitive to change in frequent words (**log**)

Pointwise Mutual Information (PMI)

Limits:

- **Very large** matrix **O(V^2)** ! Very large word vectors
- Hard to use large vectors in practice (i.e. 1M word vocabulary)
- Cannot compare word vectors estimated on 2 different corpora unless they have exactly the same vocabulary!

Idea: Build vectors with predefined size based on the PMI matrix

➔ **Dimensionality Reduction Technique**

Singular Value Decomposition (SVD)

We can decompose the PMI Matrix with SVD

1. We build a symmetric definite matrix based on the PMI
2. We decompose it ıe SVD

$$P = \mathbf{U}_d \Sigma_d \mathbf{V}_d^T$$

3. U is of size (V, d) gives us the representation of each word in a latent/embedding space

Properties of SVD:

- U is a orthonormal matrix
- U aggregates the highest variance of the original word embeddings

Limits of Dimensionality Reduction Approach

- Need to store a matrix of size O(V^2)
- SVD is O(V*d^2)

➔ It is inefficient to build a very large matrix for reducing:
Can we do both simultaneously?

Solution: Prediction-Based Word Embedding Approaches

Prediction-Based Model

Idea:

- Learn directly **dense word vectors**
- Using the ***distributional hypothesis***
- **Implicitly**, by **parameterizing words as dense vectors**
- and **learning to predict context** using this parametrization

Many word embedding methods use these ideas successfully

We present the ***word2vec skip-gram*** model (one of the most popular)

Word2Vec Skip-Gram Model

For each Sentence

1. Sample ***a target word***
2. Predict ***context words*** defined as words in a fixed window from the target word

my dog is barking and chasing its tail

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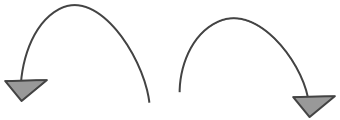

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Word2Vec Skip-Gram Model

Given $d \in \mathbb{N}$, let $\mathbf{W} \in \mathbb{R}^{(V,d)}$ and $\mathbf{C} \in \mathbb{R}^{(V,d)}$ two word representations (or word *embedding*) matrices. For each sequence $(w_1, .., w_T)$:

- Pick a *focus* word w, associated to the vector $\mathbf{w} \in \mathbb{R}^d$ ($\mathbf{w}$ is the row associated to w in $\mathbf{W}$)
- Pick a *context* word c, associated to the vector $\mathbf{c} \in \mathbb{R}^d$ ($\mathbf{c}$ is the row associated to c in $\mathbf{C}$)
- Maximize $\max_{\mathbf{W} \in \mathbb{R}^{(V,d)}, \mathbf{C} \in \mathbb{R}^{(V,d)}} log\, p(c|w)$ (*maximum likelihood estimator*)

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Word2Vec Skip-Gram Model

1. How to define $log\, p(c|w)$?
2. How to optimize $log\, p(c|w)$?

Word2Vec Skip-Gram Model

1. **How to define** $log\, p(c|w)$ **?**

2. **How to optimize** $log\, p(c|w)$ **?**

Intuition

- This is a classification problem
- The labels we want to predict are the context words
- Classification with a very large number of labels (V~100K)

Ideas:

→ **Softmax**

→ Simplify the softmax with **Negative Sampling** for Efficiency

Word2Vec Skip-Gram Model

Softmax of dot-products context vs. words vectors:

$$p(c|w) = \frac{e^{\mathbf{w}.\mathbf{c}}}{\sum_{\mathbf{v}} e^{\mathbf{w}.\mathbf{v}}}$$

We compute the log-likelihood, **our objective function**, as:

$$log\, p(c|w) = \mathbf{w}.\mathbf{c} - log \sum_{\mathbf{v}} e^{\mathbf{w}.\mathbf{v}}$$

Limits: O(V) to compute the loss (at every iteration)

→ **Negative Sampling**

Word2Vec Skip-Gram Model: Negative Sampling

Idea: Instead of computing the probability objective over the entire vocabulary (all the ***V-1*** negative context words)

→ We sample ***K words that are not in the context of w*** $v \in N_K$ *(K<<V)*

Word2Vec Skip-Gram Model: Negative Sampling

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New objective function:

$$\sigma(\mathbf{w}, \mathbf{c}) + \frac{1}{K} \sum_{v \in N_K} log\, \sigma(\text{-}\mathbf{w}, \mathbf{v}) \quad \text{with} \quad \sigma(\mathbf{x}, \mathbf{y}) = \frac{1}{1 + e^{-\mathbf{x}.\mathbf{y}}}$$

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Complexity?

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→ **O(K) to compute** with K independent of V

Word2Vec Model: Optimization

Algorithm 1 Skip-Gram Word2vec Training

Given a corpus C, made of a set of unique tokens V. Hyperparameters: number of negative samples K, a window size l, dimension of word vectors d, learning rate (α_t)

Initalize Randomly: $\mathbf{W} \in \mathbb{R}^{(V,d)}$ and $\mathbf{C} \in \mathbb{R}^{(V,d)}$

for step t in $0..T$ **do**

    ### Step 1: Sampling
    Sample $s = (w_1, .., w_n) \in C$ # a sequence in your corpus (e.g. sentence)
    Sample a pair $(i, j) \in [1, .., n]$ with $|i - j| \leq l$
    we note $w = w_i$, $c = w_j$ represented by vectors **w** in **W** and **c** in **C**
    Sample $N_K = \{v_1, .., v_K\} \subset V$ represented by $\{\mathbf{v}_1, .., \mathbf{v}_K\}$ in **C** # Negative samples
    ### Step 2: Compute loss
    $l(\mathbf{W}, \mathbf{C}) = -\sigma(\mathbf{w}, \mathbf{c}) - \frac{1}{K} \sum_{v \in N_K} log\, \sigma(\mathbf{-w}, \mathbf{v})$
    ### Step 3: Parameter update with SGD
    $\mathbf{W}_t = \mathbf{W}_{t-1} - \alpha_t . \nabla\, l(\mathbf{W}_{t-1}, \mathbf{C}_{t-1})$
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Word2Vec Model: Optimization

Loop over the dataset E times (number of epochs)

Complexity: O(d*K*T)

- ➔ **No Memory bottleneck**
- ➔ **Scale to Billion-tokens datasets**

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 $l(\mathbf{W}, \mathbf{C}) = -\sigma(\mathbf{w}, \mathbf{c}) - \frac{1}{K} \sum_{v \in N_K} log\, \sigma(\mathbf{-w}, \mathbf{v})$

 ### Step 3: Parameter update with SGD

 $\mathbf{W}_t = \mathbf{W}_{t-1} - \alpha_t.\nabla\, l(\mathbf{W}_{t-1}, \mathbf{C}_{t-1})$

 $\mathbf{C}_t = \mathbf{C}_{t-1} - \alpha_t.\nabla\, l(\mathbf{W}_{t-1}, \mathbf{C}_{t-1})$

end

Word2Vec Skip-Gram Model & the PMI

(Levy & Goldberg 2014) showed that

- Estimating the embedding matrix with Skip-Gram and Negative Sampling (SGNS)…
- …is equivalent to computing the shifted-PMI matrix

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$$M_{ij}^{\text{SGNS}} = W_i \cdot C_j = \vec{w}_i \cdot \vec{c}_j = PMI(w_i, c_j) - \log k$$

Word2Vec

- Still very popular in practice
- Works very well with Deep Learning architecture (e.g. LSTM models) to model specific tasks (e.g. NER)
- Recently "beaten" by contextualized approaches (BERT)

Extensions

- Lots of variant of the Skip-Gram exists (CBOW, Glove…)
- Multilingual setting: build shared representations across languages (fastext)

Limits

- Doesn't model morphology
- **Fixed Vocabulary**: What if we add new tokens in the vocabulary?
- **Polysemy**: Each token has a unique representation (e.g. cherry)

Evaluation of Word Embeddings

How to evaluate the quality of word embeddings?

Extrinsic Evaluation

- Use them in a task-specific model and measure the performance on your task (cf. lecture 5 & 6)

Intrinsic Evaluation

➔ **Idea:** ***"similar" words should have similar vectors***

What do we mean by "similar" words?

- Morphologically similar: e.g. *computer, computers*
- Syntactically similar: e.g. determiners
- Semantically similar: e.g. *animal* , *cat*

Intrinsic Evaluation of Word Embeddings

How to evaluate the quality of word embeddings?

Qualitative Evaluation

- Visualize word embedding space
- Case by case: look at nearest neighbors of given words

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Quantitative Evaluation

- Is **Word embedding similarity** related with **human judgment**?

👉 notebook

Intrinsic Evaluation of Word Embeddings

Visualization

Word Vectors are high dimensions (usually ~100)

- ➔ **Project the word embedding vectors** using PCA or T-SNE
- ➔ **Visualize** in **2D or 3D**
- ➔ **Analyse the clusters**

Intrinsic Evaluation of Word Embeddings

👉 notebook

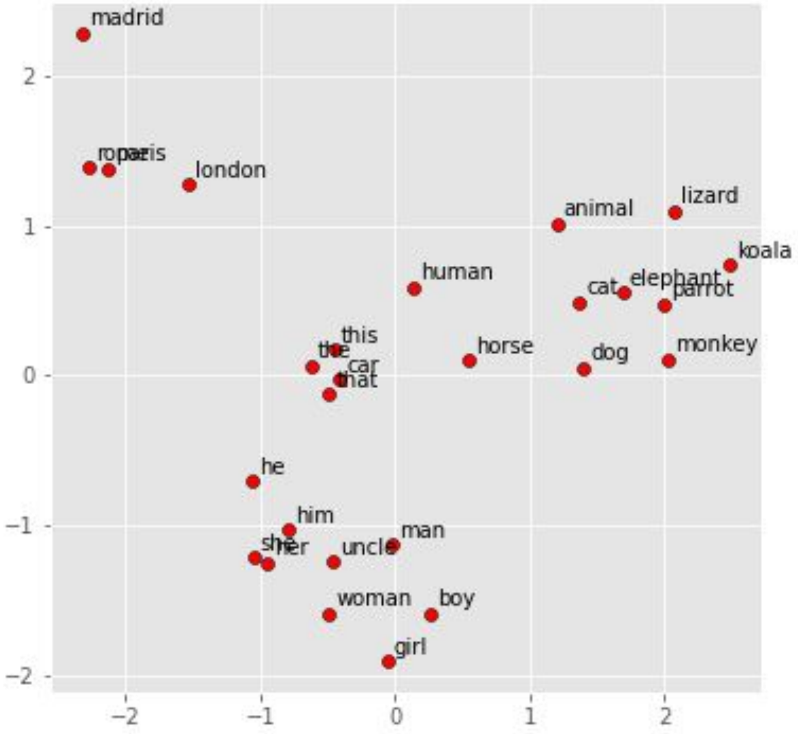

Figure: Visualize skip-gram trained on Wikipedia (1B tokens) (fastext.cc) vectors with PCA

Intrinsic Evaluation of Word Embeddings

How to measure similarity in the word embedding space?

- **Cosine Similarity**

$$sim(w_1, w_2) = cos(x_{w_1}, x_{w_2}) = \frac{x_{w_1}^T x_{w_2}}{||x_{w_1}|| \, ||x_{w_2}||}$$

- **L2 Distance**

$$sim(w_1, w_2) = L2(x_{w_1}, x_{w_2}) = ||x_{w_1} - x_{w_2}||$$

Intrinsic Evaluation of Word Embeddings

Nearest-Neighbor with the cosine similarity (skip-gram trained on Wikipedia (1B tokens))

moon	score
mars	0.615
moons	0.611
lunar	0.602
sun	0.602
venus	0.583

talking	score
discussing	0.663
telling	0.657
joking	0.632
thinking	0.627
talked	0.624

blue	score
red	0.704
yellow	0.677
purple	0.676
green	0.655
pink	0.612

Intrinsic Evaluation of Word Embeddings

We can **compare the similarity between words** in **the embedding space with human judgment**

1. **Collect Human Judgment** (or download dataset e.g. WordSim353) on a list of pairs of words
2. **Compute similarity** of the **word vectors** of those pairs
3. **Measure correlation** between both

Word 1	Word 2	Word2vec Cosine Similarity	Human Judgment
tiger	tiger	1.0	10
dollar	buck	0.3065	9.22
dollar	profit	0.3420	7.38
smart	stupid	0.4128	5.81

Application of Word Embeddings

- Downstream Tasks (Lecture 5 and 6)
- **Word Sense Induction**
- **Semantic analysis** (semantic shift in time, across communities...)

Representing Documents With Vectors

Representing Documents into Vectors

Similarly to what we saw for word-level representation we can **represent documents into vectors**

1. **Using word vectors**
2. **Count-Based Representations**
3. **Generative Probabilistic Graphical Model (e.g. LDA seen in the *lab*)**
4. **Using language models**

Representation of documents based on words

Based on word vectors representing sentence/document with vector can be done in a straightforward way with:

➔ Given sequence of word represented by x1, .., xn, define **f: → R**

$$[x_1,.., x_n] \rightarrow f(x_1,..,x_n)$$

For instance:

$$[x_1,.., x_n] \rightarrow \frac{1}{n}\sum_i x_i$$

Count-Based Representation of Documents

Given a Corpus made of novels of Shakespeare (Macbeth, Hamlet…), each document is a novel here:

1. Get the vocabulary of the Corpus
2. Compute the **Count-Based Matrix at the document-level**

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Build the ***term-frequency*** matrix

$$tf_{t,d} = |\{t \in d\}|$$

Count-Based Representation of Documents

Given a Corpus made of novels of Shakespeare (Macbeth, Hamlet…), each document is a novel here:

1. Get the vocabulary of the Corpus
2. Compute the **Count-Based Matrix at the document-level**

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Count-Based Representation of Documents

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
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mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

➔ We get a vector representation for each document of the corpus

NB: such a model is called a ***bag-of-word model*** because the ordering of the words in each document does not matter

Count-Based Representation of Documents

Limits: High sensitivity to frequent words OR to very infrequent words

How to improve?

- **A word that is in all documents** of the corpus (e.g. "the") is **not informative** at all for the document representation, still it impacts the document vector
- **A word that is in only 1 document** is likely to be **very informative** of the document

Solution:

- **Weight** the count with

➔ **Inverse Document Frequency**

Count-Based Representation of Documents

Weighting the importance of each term with the *document frequency*

Definition: *Given **N the total number of documents, a term t (token),***

$$idf_{t,C} = log(\frac{|C|}{|\{d \in C, s.t. \ t \in d\}|})$$

NB: Compute the *log* to smooth the impact of words that are in only a few documents

TF-IDF Representation of Documents

Matrix becomes: *tf*idf(t,d,C)*

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

TF-IDF Representation of Documents

We can then apply dimension reduction technique to get dense vectors

➔ E.g. we can apply SVD: **Latent Semantic Analysis**

Session Summary: Representing text with Vectors

1. **Word as 1-hot vectors (// or indexes)**

2. **Hand-Crafted approach (e.g. Wordnet)**

Word Vectors inferred with data using the distributional hypothesis:

3. **Word vectors with count-based approach**

4. **Prediction-Based Approach with the skip-gram model**

5. **Document Representation: bag of word models and the tf-idf**

Bibliography and Acknowledgment

- Distributional Semantics, Evert 2015
- Foundations of Distributional Semantics, Evert & Lenci 2009
- Stanford Information-Retrieval Book
- Miller et. al 1985: WordNet
- Mikolov et. al 2013: Word2vec
- Grave & Joulin, ENSAE 2018-2019