

Language Modeling

Machine Learning for Natural Language Processing, ENSAE 2022

Lecture 4

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Lectures Outline

1. The Basics of Natural Language Processing (February 1st)
2. Representing Text with Vectors (February 1st)
3. Deep Learning Methods for NLP (February 8th)
4. **Language Modeling (February 8/15th)**
5. Sequence Labelling (Sequence Classification) (February 15th)
6. Sequence Generation Tasks (February 15th)

Outline

- **Causal Language Model with LSTM**
- **Causal Language Model with Transformers**
- **Evaluation**

Framework

Given $(t_1, \dots, t_D) \in V^D$, our goal is to estimate: $P(t_{n+1} | t_1, \dots, t_n)$

We saw how to estimate that with n-gram models

To do better:

→ Use a Deep-Learning Model

Why Deep Learning Models for LM?

Motivations

Theoretical Insights

- Deep Learning Models are **universal approximators**
- Recurrent Neural Network can in theory model infinite context

Practical Insights

- They can be trained on **very large amount of data**
- They can use **continuous representation** of input tokens capturing the *distributional hypothesis* efficiently

Framework

Given $(t_1, \dots, t_D) \in V^D$, **our goal is to estimate:**

$$p(t_{n+1} | t_1, \dots, t_n)$$

Framework

We want to find dnn_{θ}

$$dnn_{\theta} : \quad V^D \quad \rightarrow \quad [0, 1]^V$$

$$(t_1, \dots, t_D) \mapsto \hat{p}$$

s.t. $\hat{p} = (p_i)_{i \in [0, V-1]}$, $\forall i \quad p_i \in [0, 1]$ and $\sum_i p_i = 1$

Design Questions

- ★ **What tokenization ?**
- ★ **What output activation function and loss?**
- ★ **What architecture?**
- ★ **How do you represent a token to feed the model?**

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NB: Questions to ask for any NLP task approached with Deep Learning

Tokenization

- **Word-Level Tokenization: e.g. “*I, am, going*”**

Pros: Easy to segment, Words are Linguistic Units

Cons: Out-of-Vocabulary (OOV) problem

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Pros: No OOV problem

Cons: Very long sequences

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- **SentencePiece Tokenization:** *“_I, _am, _go, ing”*

Frequent “words” become tokens and infrequent ones are split into subwords

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Frequent “words” are kept intact and infrequent ones are split into subwords

NB: SentencePiece is the most popular tokenization algorithm for language models

Output Activation & Loss

Softmax Function

$$\text{softmax}(s) = \left(\frac{e^{s_i}}{\sum_k e^{s_k}} \right)_{i \in [1, V]}, \text{ for } s \in \mathbb{R}^V.$$

Loss Function

$$l(p, \hat{p}) = CE(p, \hat{p}) = \sum_{i \in [0, V-1]} p_i \log(\hat{p}_i)$$

Output Activation & Loss

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$$\text{softmax}(s) = \left(\frac{e^{s_i}}{\sum_k e^{s_k}} \right)_{i \in \llbracket 1, K \rrbracket}, \text{ for } s \in \mathbb{R}^K$$

Loss Function

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NB: We will use them in all the tasks we will cover in this course

Architecture

- The Multi-Layer Perceptron
- Recurrent Neural Network: LSTM Model
- The Transformer

MLP for Language Modeling

Recall: The **MLP** works **on unidimensional data** (e.g. dimension d)

$$\begin{aligned} dnn_{\theta} : \quad \mathbb{R}^d &\rightarrow [0, 1]^V \\ X &\mapsto \hat{Y} \end{aligned}$$

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→ Truncate input sequences: **Fixed-Window Language Modeling**

How to represent input tokens?

Solution 1

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1. 1-Hot Encoding

How to represent input tokens?

Solution 1: 1-Hot Encoding

1. We associate each *token* to a **1-hot vector of size D**

movie = $[1, 0, \dots, 0, 0, 0]$

hotel = $[0, 1, \dots, 0, 0, 0]$

...

art = $[0, 0, \dots, 0, 0, 1]$

2. **Concatenate them** to get a unidimensional vector

1-Hot Encoding as inputs

$$dnn_{\theta} : \{0, 1\}^{|V|*K} \rightarrow [0, 1]^V$$

$$x = ([x_1, \dots, x_K]) \mapsto \hat{p}$$

- First hidden layer is of size $|V|*K$
- Taking as input **a sparse vector**

1-Hot Encoding as inputs

$$dnn_{\theta} : \{0, 1\}^{|V|*K} \rightarrow [0, 1]^V$$

$$x = ([x_1, \dots, x_K]) \mapsto \hat{p}$$

First hidden layer:

assuming tanh as the activation function, dimension δ

$$h_1 = \tanh(W.x) \text{ s.t. } W \in \mathbb{R}^{\delta \times (|V|*K)}$$

1-Hot Encoding as inputs

$$dnn_{\theta} : \{0, 1\}^{|V|*K} \rightarrow [0, 1]^V$$

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Limits

- The representation of each token is fixed and a 1-hot vector
- In this approach, **we do not learn** a representation of **each input token**

How to represent input tokens?

Solution 2: Integrate an Dense Embedding Layer

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We define a dense embedding layer $E \in \mathbb{R}^{\delta_e \times |V|}$.

This means that for each token $t \in V$ indexed by j in the vocabulary $V = \{t_1, \dots, t_{|V|}\}$ we have t_j *embedded* by the vector $E_{.j}$ (i.e. column of the matrix E indexed by j) of dimension δ_e (the dimension of the embedding vectors).

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- We can train it during **backprop end-to-end**

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[See how to define it in torch](#)

Dense Embedding Layer

$$dnn_{\theta} : \mathbb{R}^{K * \delta_e} \rightarrow [0, 1]^V$$

$$x = ([x_1, \dots, x_K]) \mapsto \hat{p}$$

s.t. $x_i = E_{.j} \in \mathbb{R}^{\delta_e}$ with token t_i indexed by j in V

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s.t. $x_i = E_{.j} \in \mathbb{R}^{\delta_e}$ with token t_i indexed by j in V

- E is a dense embedding matrix
- We can **learn a representation vector for each token in the vocabulary**

Why is an Embedding Layer much better?

Trainable Dense Embedding layers are a **“game changer”** for Deep Learning Models in NLP i.e. Generalization is much better compared to 1-hot encoding

Why?

(intuition)

t and t' that have the embedding vectors (in E) x and x' . e.g. $t = \text{"dog"}$ and $t' = \text{"cat"}$

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How to initialize an Embedding Layer?

Similarly to all other parameters in a deep learning model

- Before starting training: **we can simply initialize the embedding matrix randomly**
- Before training, the similarity between **embedding word vectors is random**

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Can we do better?

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- Before starting training: **we can simply initialize the embedding matrix randomly**
- Before training, the similarity between **embedding word vectors between random**

Can we do better?

- In lecture 2 we have seen how to represent good dense embedding vector with skip-gram word2vec model
- **We can simply initialize our word embedding matrix with word2vec vectors**

How to initialize an Embedding Layer?

Initializing with a **pretrained embedding** layer was also a **gamechanger** for many NLP tasks and many Deep Learning architecture

Conditions to use a pretrained embedding layer:

- The token in our vocabulary must be in the training of the word2vec model
- For the one that were not seen, we can simply initialize them randomly

Transfer Learning in NLP

Initializing with a **pretrained embedding** layer is also a ***game changer*** for many NLP tasks and many Deep Learning architecture

It is called **Transfer Learning**

Embedding Layer Summary

- Trainable Dense Embedding Layer are a *game changer* for Deep Learning Models
- Even more when we can use a pretrained embedding layers (e.g. with word2vec)
- They can be used with all Deep Learning Architectures
- **For all NLP tasks**

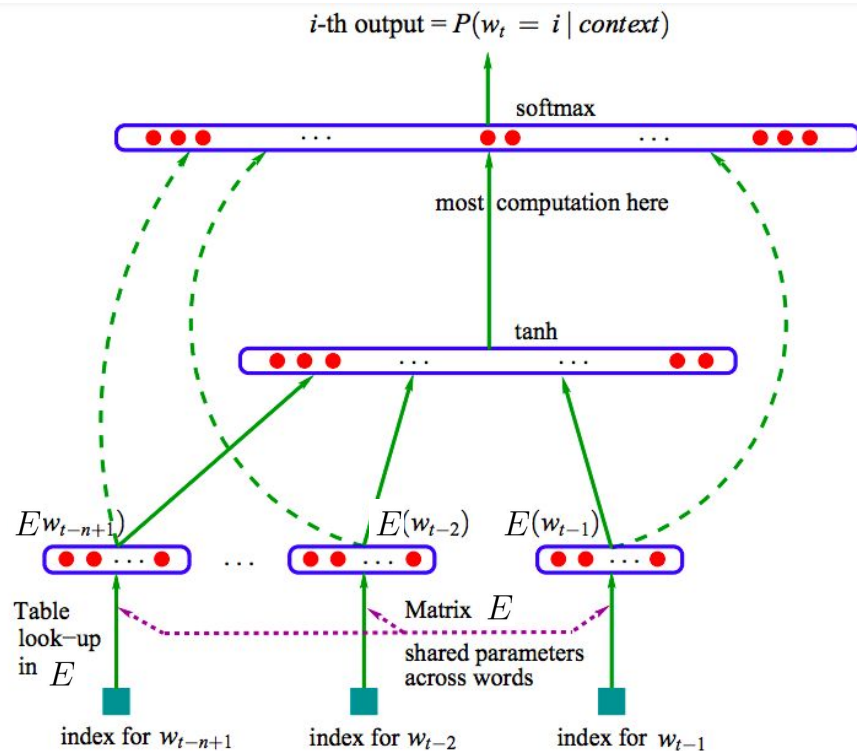
MLP for Fixed-Window Language Modeling

$$\hat{t}_{n+1} = \operatorname{argmax}_{t \in V} p(t|t_1, \dots, t_n)$$

$$dnn_{\theta} : \mathbb{R}^{|V| * \delta_e} \rightarrow [0, 1]^V$$

$$x = ([x_1, \dots, x_K]) \mapsto \hat{p}$$

MLP for Fixed-Window Language Modeling



Limits of MLP for language modeling

- *Windows is Fixed*
- Use Recurrent Neural Network (e.g. LSTM)

Recurrent Neural Network for LM

Recall:

$$h_{i+1,t+1} = \varphi_i(W_i h_{i,t} + U_i h_{i+1,t} + b_i), \forall i \in [1, L - 1]$$

with $h_{1,t} = X_t$ and $\hat{Y}_t = \text{dnn}(X_t) = h_{L,t} \forall t \in [1, T - 1]$

Recurrent Neural Network for LM

Recall:

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with $h_{1,t} = X_t$ and $\hat{Y}_t = \text{dnn}(X_t) = h_{L,t} \forall t \in [1, T - 1]$

For Language Modeling , like we did for the MLP

- **We use an Embedding layer**
- **We use the softmax layer as output**

Recurrent Neural Network for LM

For an sequence of token

$$h_{i+1,t+1} = \varphi_i(W_i h_{i,t} + U_i h_{i+1,t} + b_i), \forall i \in [1, L] \forall t \in [1, T]$$

with $h_{1,t} = Emb(x_t)$ and $p_{t+1}^{\hat{}} = h_{L+1,t+1}$

with $\varphi_L = softmax$

We estimate

$$p_{t+1}^{\hat{}} = p(x_{t+1} | x_1, \dots, x_t)$$

directly with the RNN

Recurrent Neural Network for LM

Written in a more synthetic way

$$h_{i+1,t+1} = RNN_i(h_{i,t}, h_{i+1,t}), \forall i \in [1, L] \forall t \in [1, T]$$

$$\text{with } h_{1,t} = Emb(x_t) \text{ and } p_{t+1}^{\hat{}} = h_{L+1,t+1}$$

$$\text{with } \varphi_L = softmax$$

We estimate

$$p_{t+1}^{\hat{}} = p(x_{t+1} | x_1, \dots, x_t)$$

directly with the RNN

Recurrent Neural Network for LM

With a LSTM, we have a dependency on the *Cell Vector*:

$$h_{i+1,t+1}, C_{i+1,t+1} = LSTM_i(h_{i,t}, h_{i+1,t}, C_{i+1,t}), \forall i \in [1, L] \forall t \in [1, T]$$

with $h_{1,t} = Emb(x_t)$ and $p_{t+1} = h_{L+1,t+1}$

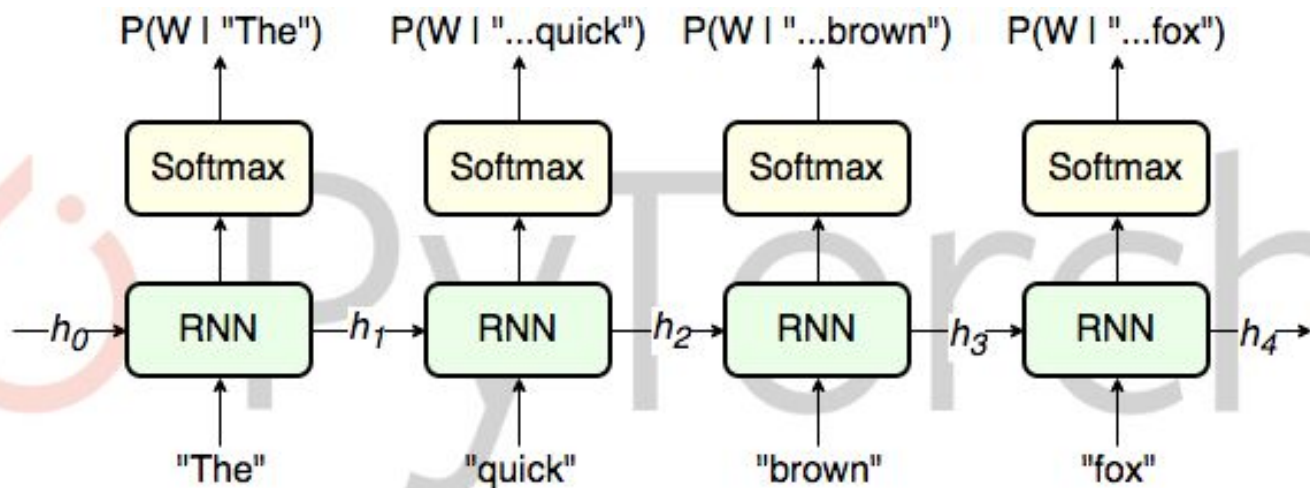
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We estimate

$$p_{t+1}^{\hat{}} = p(x_{t+1} | x_1, \dots, x_t)$$

directly with the LSTM

Recurrent Neural Network for LM



Transformer for Causal Language Modeling

Inputs: Transformers requires a fixed sequence at input (we note it \mathcal{T})

Let's assume we have a sequence (x_1, \dots, x_T)

We simply append it with a **PADDING** token

We append $(x_{T+1}, \dots, x_{\mathcal{T}})$ with $x_t = [PAD] \forall t \geq T + 1$

We get a sequence of length \mathcal{T} : $(x_1, \dots, x_{\mathcal{T}})$

We make the model ignore those tokens by setting the softmax scores to 0 in the self-attention

Transformer for Causal Language Modeling

Input

$$(x_1, \dots, x_{\mathcal{T}})$$

Embeddings:

Embedding:

$$(Emb(x_1), \dots, Emb(x_{\mathcal{T}}))$$

such that $Emb(x_i) = PositionEmb(x_i) + TokenEmb(x_i)$

Transformer for Causal Language Modeling

Given a sequence of tokens: (x_1, \dots, x_T)

$$H_{i+1} = \text{FeedForward}(A_{i+1}) \text{ and } A_{i+1} = \text{SelfAttention}(H_i) \quad \forall i \in [1, L]$$

$$\text{with } \text{SelfAttention}(H_i) = \text{softmax}\left(\frac{QK^T}{\sqrt{\delta_K}}\right)V$$

$$H_0 = (\text{Emb}(x_1), \dots, \text{Emb}(x_T))$$

Transformer for Causal Language Modeling

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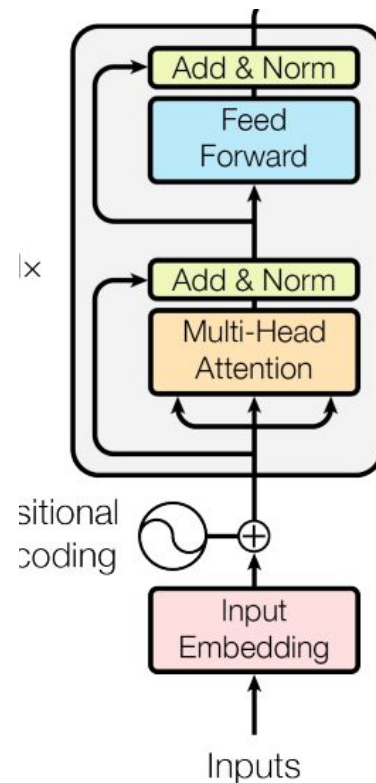
$$H_0 = (\text{Emb}(x_1), \dots, \text{Emb}(x_T))$$

- **Residual Connection** and **Layer Norm** are not included in those equations
- **FeedForward is position-wise** two layer MLP (i.e. applied independently from the position of each hidden vector)
- Self-Attention is actually a **Multi-Head Self-Attention**

The Transformer Architecture

The Transformer Architecture is

- Stack of [Self-Attention + FF Layer]
- With Skip-Layer and Normalization in between
- Encoding the position with positional vector



Transformer for Causal Language Modeling

Given a sequence of tokens: (x_1, \dots, x_T)

⇒ **Last element of the sequence of the hidden states** of the last layer fed to a softmax

$$p_{x_{T+1}^{\hat{}}} = \text{softmax}(h_T) \quad \forall t \leq T$$

Training

- We train on large corpus of text (+1G of text)
- We train them with backpropagation
- We usually do “**teacher-forcing**”, for each step, we use the “**gold**” sequence” and not the predicted one
- For Transformers, we train on **sequences as long as possible** (~1000 tokens)

Evaluation

$$\text{perplexity}(\hat{x}, x) = 2^{-\sum_i x_i \log(\hat{x}_i)}$$

The lower the perplexity the better the language model

Empirical Performance

Language Model Performance Comparison

→ **Transformer Models outperform LSTM-based models**

Lecture Summary

- **Causal Language Modeling Framework**
- **Representing input tokens for language modeling**
- **Recurrent Neural Network for Language Modeling**
- **Transformer for Language Modeling**